Model-based Feature Extraction of Electrocardiogram Using Mean Shift

Jingyu YAN Student Member IEEE, Yan LU, Jia LIU, Xinyu WU and Yangsheng XU Fellow IEEE

Abstract—Feature extraction of electrocardiogram (ECG) is the fundamental work of further automatic diagnosis. However, suffered from various kinds of noises with white, pink, and other colors, feature extraction is not a straightforward work but requires necessary signal processing techniques. In this paper, we propose an accurate and robust ECG feature extraction method based on mean shift algorithm which has the ability to remove noise involved in input signal by taking advantage of its embedded Gaussian filter and locate extremes of input signal using gradient optimization based on self-adaptive search steps. To demonstrate the availability and efficacy of the proposed method, we conduct experiments on signals contaminated by noises of white, pink and brown colors from 5dB to 15dB signal-to-noise ratios. Clean signals are produced by ECG synthesizer (ECGSyn) so that we can obtain the real features and quantitatively calculate feature extraction errors of the proposed method. Experiment results verify that our method can handle various kinds of noises and achieve satisfactory feature extraction performance.

I. INTRODUCTION

The electrocardiogram (ECG or EKG) is a record of bioelectric potential produced by the rhythmic human heart activities, i.e. contraction and relaxation. Since ECG provides valuable information of heart functional conditions, automatic ECG feature extraction is of high importance for further clinical analysis of heart health. Recently, various methods have been proposed to extract ECG features from time domain, e.g. QRS complexes detection [1], to frequency domain, e.g. power spectral density estimate [2].

Since a typical ECG beat is characterized by P, Q, R, S, and T waves, in this paper, we can adopt the location, amplitude and width of each wave as the ECG features, just like the usual clinician usage of ECG signals. Although there have been many studies on extracting some of these features, for example, Last et al utilized wave component templates to identify wave features [3] and Cuiwei et al developed an algorithm based on wavelet transform for detecting ECG characteristic points [4], these methods are usually lack of quantitative performance analysis based on various kinds of noises, such as pink and brown noises.

In this paper, we propose a mean shift based approach for ECG feature detection with a robust performance to kinds of noisy signals. Section II introduces the basic principle and procedure of mean shift algorithm and proposes enhancement strategies to improve performance. Section III illustrates the implementation of feature extraction method based on enhanced mean shift algorithm. Experiments and qualitative analysis are conducted in section IV. Conclusions and future works are provided in section V.

II. ENHANCED MEAN SHIFT ALGORITHM

Originally, mean shift algorithm is proposed to locate stationary points of a probability density function given a set of data sampled from this distribution. However, if we treat a normalized sequence of discrete signal as a set of samples from a certain probability distribution and the signal indexes as distinct sample values, and regard each signal value as the sample multiplicity (i.e. the number of times one datum has been sampled), mean shift technique can be applied to find extreme points of the envelope of given signal. As mean shift algorithm is derived from statistics, it also involves an embedded kernel filter, realized by Gaussian function in this work.

A. Principle and Procedure

The first step is to convert the original discrete sequence \( S \), with length \( N \), to a continuous signal \( f(x) \) using \( \delta \) interpolation.

\[
 f(x) = \sum_{k=1}^{N} S(k) \delta(x - k) \quad \delta(x) = \begin{cases} 1 & (x = 0) \\ 0 & (\text{others}) \end{cases} \quad (1)
\]

Since \( f(x) \) is non-derivable and completely reserves the noise involved in original data, we convolute it with the kernel Gaussian function and adopt the convolution as a smoothed version \( \tilde{f}(x) \).

\[
 \tilde{f}(x) = f(x) * G_\sigma = \sum_{k=1}^{N} S(k) \delta(x - k) * G_\sigma(x,k) = \sum_{k=1}^{N} S(k) \times G_\sigma(x,k) \quad (2)
\]

where \( G_\sigma(x,\mu) \) stands for the Gaussian kernel with mean \( \mu \) and standard deviation \( \sigma \). Clearly, the gradient function of \( \tilde{f}(x) \), denoted as \( g(x) \), is given by

\[
 g(x) = \frac{df(x)}{dx} = \sum_{k=1}^{N} S(k) \frac{dG_\sigma(x,k)}{dx} = \frac{1}{\sigma^\sqrt{2\pi}} \left( \sum_{k=1}^{N} S(k) \times k \times e^{-\frac{(x-k)^2}{2\sigma^2}} - \sum_{k=1}^{N} S(k) \times x \times e^{-\frac{(x-k)^2}{2\sigma^2}} \right) \quad (3)
\]

Without loss of generality, we next apply the gradient-based iteration method to find local maxima of \( \tilde{f}(x) \) (Since the procedure to find local minima is similar, we neglect its introduction limited by paper length). Given iteration step

Jingyu YAN, Yan LU and Yangsheng XU are with Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong, China jyyuan,ylu,yxwu@mas.cuhk.edu.hk
Jia LIU and Xinyu WU are with Shenzhen Institute of Advanced Technology, China jia.liu,xy.wu@siat.ac.cn
equation (4) can be simplified to be

\[ x(i + 1) = x(i) + \text{step}(i) \times g(x(i)) \]  (4)

To simplify the equation, we set \( \text{step}(i) \) to be

\[ \text{step}(i) = \frac{\sigma^3 \times \sqrt{2\pi}}{\sum_{k=1}^{N} S(k) \times e^{-\frac{(x(i) - k)^2}{2\sigma^2}}} \]  (5)

Obviously, \( \text{step}(i) > 0 \), for \( S \) has been normalized. Then, equation (4) can be simplified to be

\[ x(i + 1) = \frac{\sum_{k=1}^{N} S(k) \times k \times e^{-\frac{(x(i) - k)^2}{2\sigma^2}}}{\sum_{k=1}^{N} S(k) \times e^{-\frac{(x(i) - k)^2}{2\sigma^2}}} \]  (6)

This result is intuitive: the local mean is shifted towards the region where the majority of the data resides, and the vector \( v(i) = x(i + 1) - x(i) \) is thus called mean shift vector.

Using a signal index \( k \) (\( k = 1, 2, \cdots, N \)) as initial search point \( x(0) \), the algorithm iterates mean shift process according to equation (6), until the norm of mean shift vector \( v(i) \) is below a threshold \( \text{StopDist} \) or the iteration number reaches the predetermined maximum \( \text{MaxIter} \). Then, the final value of \( x(i) \) is considered as the location of the local maximum (or minimum) point of the signal corresponding to the initial index \( k \), denoted as \( p(k) \).

Since it is the Gaussian kernel which facilitates the mean shift algorithm to suppress noises which may produce fake local extremes, the algorithm performance strongly depends on the value of \( \sigma \) in the Gaussian function. A suitable selection of \( \sigma \) can assure a desirable configuration of \( f(x) \), such that the found local extremes match the waves of the clean signal well. However, if \( \sigma \) is too small or too large, \( f(x) \) will take on undesirable profiles and thus result in false extremes, as shown in Fig. 1. A self-adaptive selection of \( \sigma \) will be introduced later.

**B. Enhancement Strategy**

To enhance the performance of the mean shift algorithm, we propose two enhancement strategies to deal with the following issues: (1) the instinct problem that the algorithm underestimates the end point values of the signal sequence, and (2) how to find all the local extremes of the signal in a timesaving way.

1) Segment expansion

According to the mean shift algorithm, the value of \( f(x^*) \) is calculated depending on the values of neighboring elements, weighted by their distances to \( x^* \). Since the data sequence has limited length, if \( x^* \) is the start or terminal point, its left or right neighboring elements are all counted as 0, the smallest value in the normalized data sequence. Therefore, underestimate can not be avoided at the end points, sometimes causing unexpected fake extremes.

Segment expansion strategy is proposed to correct the underestimate problem by assigning the left-neighboring elements of the start point the same value as the start point and the right-neighboring elements of the terminal point the same value as the terminal point. In this case, the data segment is expanded as:

\[
\underbrace{s(1), \cdots, s(1)}_{\text{Left Expansion}} \begin{array}{c} \underbrace{s(1), s(2), \cdots, s(N),}_{\text{Original Segment}} \underbrace{s(N), \cdots, s(N)}_{\text{Right Expansion}} \end{array}
\]

It is necessary to note that the extreme searching is confined within the original region and the expanded data are only used for calculating \( f(x) \). The length of expanded region can be set as \( 3 \times \sigma \), which accounts for about 99.7% of the Gaussian window.

2) Time-efficient extreme detection method

As discussed above, given a signal element at index \( k \), its corresponding local extreme can be found through the mean shift iteration. Since we need to obtain all the local extremes from the signal sequence, a natural way is to list the local extreme corresponding to each signal element one by one. However, this process would be painfully time consuming, especially when we have a long signal sequence. Considering that we need find only several extremes which are expected to correspond to the waves of the ECG signal, a strategy base on self-adaptive search step is proposed to accelerate the search process as follows.

Suppose now we have got the local extreme’s location \( p(k) \) through a mean shift iteration started at index \( k \). Then, the next iteration process will not start from \( k+1 \), but from \( k+\text{jump} \) by jumping a large step, where \( \text{jump} = N/5 \) by experience. If the difference between \( p(k+\text{jump}) \) and \( p(k) \) is below a threshold \( \text{MinDist} \), then they are merged as their mean. Otherwise, let \( \text{jump} = \text{jump}/2 \) and start the iteration from \( k+\text{jump} \) again, until \( p(k+\text{jump}) \) and \( p(k) \) are close enough or \( \text{jump} \) reduces to 1.

**III. ECG Feature Extraction Based on Mean Shift Algorithm**

Traditionally, ECG features are extracted based on a heart beat consisting of P, Q, R, S, T waves in sequence.
Although this representation of heart beat strictly follows the physiological cycle of heart activity, a drawback involved is that the start and end of a heart beat are usually difficult to determine, for they are both in low amplitude. Since the accuracy of R wave detection has been considerably high based on some well developed methods [5][6][7], we adopt the signal segment between two consecutive R waves as a heart beat.

A. Wave segmentation

Without constraints, the found wave extremes may be strangely distributed: too close or too dispersed in some region, due to the existence of significant noise. This distribution may match the signal envelop well mathematically, but it has no physiological meaning. For example, since the duration of QRS complex is usually less than 0.12s, it is illogical to find the crests (or troughs) of Q and R waves are 0.15s apart. In other words, the crest (or trough) of each wave in a heart beat is supposed to locate within a limited period of time, so that it has a good interpretation of the actual heart activity. According to the statistics of ECG data from [8], a beat segment can be divided into six corresponding subsegments as given in Table. I.

B. Estimation of wave location

Based on the basic principle that only one extreme at most is expected in a wave subsegment, a self adaptive σ selection method is proposed to locate each wave.

Starting from $\sigma = N/10$ empirically, we firstly find all the possible extremes in a wave subsegment using enhanced mean shift algorithm. According to the number of found extremes, four situations are respectively considered as follows.

Except the two extremes at end points:

- If no other extreme is found, adopt the maximum end point as the peak of R or R’ wave, or set the extreme location as -1 for S, T, P or Q wave to represent wave disappearing in abnormal beats;
- If only one more extreme is found, it is the very desired extreme;
- If two or three more extremes are found, the one with the biggest $k$-th order difference is adopted as the desired extreme, where $k$-th order difference of point $x$ is given by:

$$\text{diff}_{ik}(x) = |2S(x) - S(x - k) - S(x + k)| \quad (7)$$

and $k$ is selected according to the typical wave duration: for R and R’ waves $k = 0.005 \times f_s$, for S and Q waves $k = 0.01 \times f_s$, for T wave $k = 0.07 \times f_s$ and for P wave $k = 0.05 \times f_s$;
- Otherwise, set $\sigma = \sigma + \Delta \sigma$, where $\Delta \sigma = N/20$ by experinece, and repeat the extreme search process until the desired extreme is found.

C. Estimation of wave amplitude

Amplitude of each wave is the height difference between each wave extreme found above and the isopotential of the whole heart beat. Three steps are taken as follows:

- Select a suitable number of points from the beat which have the smallest $k$-th order difference. In our work, the number is chosen as $N/20$, and $k = 16$.
- With the selected points, apply curve fitting to get a line as the isopotential.
- Apply the height difference between isopotential and the average height of points in the neighboring region of each extreme. The length of neighboring region is set as 5 in this paper. If the extreme position is -1, assign zero to amplitude.

D. Estimation of wave width

Based on the principle of dynamical model of ECG proposed in [9], six Gaussian functions can be applied to model the six waves in a heart beat. A heart beat can be represented as the summation of six Gaussian functions:

$$ECG(x) = \sum_{i \in \{R,S,T,P,Q,R'\}} a_i \exp \left( -\frac{(x - l_i)^2}{2w_i^2} \right) \quad (8)$$

where $x \in [1, N]$, $a_i$, $w_i$, $l_i$ respectively donate the amplitude, width and location of the $i$-th Gaussian function. If we assign the location and amplitude of each wave found above to those of the corresponding Gaussian function, then only the width $b_i$ remain to be determined. This problem can be solved by minimizing the difference between the model and the signal with respect to $b_i$, and in this work, a “lsqnonlin” [10] based optimization approach is utilized to find the values for $b_i$.

IV. EXPERIMENTS

To evaluate the efficacy of the mean shift based ECG feature extraction algorithm, experiments are conducted based on synthetic ECG signals produced by ECGSyn [9], which provides the real location and amplitude features as the baseline of error calculation. The clean signals are interrupted by white, pink, and brown noises with the Signal Noise Ratios (SNRs) 5dB, 10dB, and 15dB.

As shown in Table II, the proposed method has the ability of extracting features from ECG signals interrupted by white noise. However, since pink noises contain more low frequency components, which vary signal morphology so significantly to make some wave disappear (e.g. the P wave in 5dB pink signal) or cause extra fake waves (e.g. the wave between T and P waves in 10 dB pink signal), the errors are larger than those of white noises. The major frequencies of brown noises are lower than those of ECG signals. Therefore, brown noises chiefly vary the baseline of ECG signals rather than wave morphologies. In this case, mean shift algorithm achieves satisfactory performance of wave location, but obtains the largest amplitude errors.

As the synthetic ECG signal can not directly provide width information of each wave, we have to use the reconstructed signal using the extracted features according to the equation (8). Typical results are shown in Fig. 2. The good match of original and reconstructed signals verifies the accurate width feature extraction.
TABLE I
WAVE SUB-SEGMNTATION STRATEGY. $f_s$ IS SAMPLING FREQUENCY AND $N$ IS BEAT LENGTH.

| Color   | SNR  | Typical Beat | R       | S       | T       | P       | Q       | R’      | Location Error (ms) |
|---------|------|--------------|---------|---------|---------|---------|---------|---------|---------------------|---------------------|
| White   | 5dB  |              | 0.09    | 0.09    | 0.11    | 0.13    | 0.09    | 0.13    | 3.76                | 6.67                |
|         | 10dB |              | 0.08    | 0.04    | 0.06    | 0.10    | 0.09    | 0.08    | 3.67                | 4.03                |
|         | 15dB |              | 0.08    | 0.03    | 0.02    | 0.05    | 0.07    | 0.08    | 3.67                | 4.30                |
| Pink    | 5dB  |              | 0.12    | 0.14    | 0.21    | 0.19    | 0.14    | 0.18    | 4.00                | 8.14                |
|         | 10dB |              | 0.08    | 0.07    | 0.11    | 0.10    | 0.10    | 0.11    | 3.64                | 4.39                |
|         | 15dB |              | 0.09    | 0.03    | 0.04    | 0.07    | 0.09    | 0.10    | 3.82                | 4.23                |
| Brown   | 5dB  |              | 0.18    | 0.14    | 0.11    | 0.30    | 0.34    | 0.39    | 3.85                | 3.25                |
|         | 10dB |              | 0.10    | 0.08    | 0.05    | 0.16    | 0.19    | 0.21    | 3.91                | 2.49                |
|         | 15dB |              | 0.09    | 0.05    | 0.04    | 0.11    | 0.15    | 0.16    | 3.91                | 2.31                |

Fig. 2. Reconstructed signal based on the features extracted from signals interrupted by (a) 5dB white noise, (b) 5dB pink noise, and (c) 5dB brown noise.

V. DISCUSSION AND CONCLUSION
This paper proposed a mean shift based ECG feature extraction method. Experiments based on white, pink, and brown noise with SNR from 5dB to 15dB have demonstrated the availability and efficacy of the proposed method. However, the time consumption is comparatively high so that it can be only used as an off-line tool. In the future, how to speed up the feature extraction process becomes a possible research issue.

VI. ACKNOWLEDGMENT
This study is funded by Shenzhen Science and Technology Programme (No.SY200806300297A).

REFERENCES